ELECTROSTATICS - 1 (Second Year)

It is very important chapter not only for Intermediate but also for different competitive exams like JEE (Main & Adv.), BITSAT, EAMCET.

Electrostatics and Gravitation are the two chapters where there is a strong correlation between the concepts like Force, Field, Potential and Potential Energy. By using Conceptual Integration Approach (CIA), student can easily prepare these two topics together. This approach not only saves the time but also memory. For example the gravitational force between two point masses separated by distance r is proportional to $\frac{1}{r^2}$, just like the electric force between two point charges. In Electrostatics, the force can be attractive (–F) or repulsive (+F) but in gravitation always attractive (–F).

Electrostatics:

Charge $q = ne$ → Force $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2}$ → Field $\vec{E} = \frac{\vec{F}}{q}$ → Potential $V = -\int_{\infty}^{r}\frac{\vec{F}\cdot d\vec{r}}{q_0}$; $E = -\frac{dV}{dr}$ → Potential energy $U = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r}$

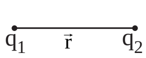
Gravitation :

Mass m → Force $F = \frac{Gm_1m_2}{r^2}\hat{r}$ → Field $\vec{g} = \frac{\vec{F}}{m}$ → Potential $V = \int\frac{\vec{F}\cdot dr}{m}$ → Potential energy $U = -\frac{Gm_1m_2}{r}$

Basic Concepts:

1. **Quantization of Charge:** Any charged body, big or small, has a total charge q which is an integral multiple of e, i.e. $q = \pm$ ne, where n is an integer having values 1, 2, 3, …, 'e' is the charge of electron which is equal to 1.6 × 10^{-19} C.

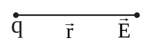
2. **Coulomb's law:** $\vec{F} = \frac{1}{4\pi\varepsilon}\cdot\frac{q_1q_2}{r^2}\hat{r}$



The force between the two point charges is directly proportional to product of the charges and inversely proportional to square of the distance between the two charges. Where $\hat{r}$ is unit vector along the line joining the two charges.

3. **Electric Field:** The space around the charge upto which the influence of charge can be felt is called electric field. Its magnitude is equal to force experienced per unit positive charge.

$\vec{E} = \frac{\vec{F}}{q_0}$, where q_0 is the unit positive charge.



Field due to a point charge:

$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2}$

4. **Electric Potential:** The work done in moving a unit positive charge from infinity to a point P inside the field is called electric potential (V).

$V = \int_{\infty}^{r}\frac{\vec{F}\cdot d\vec{r}}{q_0}$

$V = \int_{\infty}^{r}\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{q_0}dr\cos 180°$

$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$

Electric potential due to a number of point charges

$V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}$

Potential Difference:

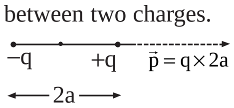
$V_2 - V_1 = -\int_{\vec{r}_1}^{\vec{r}_2}\vec{E}\cdot d\vec{r} \Rightarrow E = -\frac{dV}{dr}$

The electric field is along the direction where the potential decreases at the maximum rate.

5. **Electric Potential Energy of system of two charges:** Electric potential energy between two point charges $U = \frac{q_1q_2}{4\pi\varepsilon_0 r}$

6. **Electric Dipole:** Two point charges of equal magnitude but opposite signs separated by a small distance form an electric dipole.

7. The dipole moment of a dipole is a product of charge and distance between two charges.



Dipole moment is vector quantity,. $\vec{p} = q\times 2a$

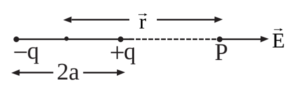
Direction of dipole moment $\vec{p}$ is from –q to +q.

Importance of Dipole:

- Dipole plays a very important role in Physics. When molecules like C_2O, CH_4 are placed in external field, they develop dipole moment. The molecules which are having dipole moment in the absence of an electric field are called polar molecules. Eg. H_2O. Various material give rise to interesting properties and important applications in the presence or absence of electric field.

- **Electric field of a dipole:**

1. **Along axial line:**



The electric field at a point P which is at a distance r from the centre of dipole is $\vec{E} = \vec{E}_+ + \vec{E}_-$

$\vec{E}_+ \to$ Electric field due to positive charge

$\vec{E}_- \to$ Electric field due to negative charge.

$\vec{E} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{p}$, where is $\hat{p}$ unit vector along the dipole axis (from –q to +q).

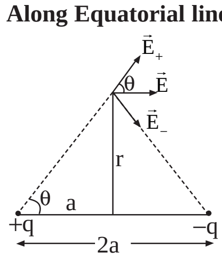
$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{4ar}{(r^2-a^2)^2}\hat{p}$

For r >> a, for a short dipole

$\vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{2\vec{P}}{r^3}$

Electric field due to dipole varies as $1/r^3$, whereas due to single charge it varies as $1/r^2$

- **Along Equatorial line:**



$\vec{E}_+ = \frac{q}{4\pi\varepsilon_0}\frac{1}{(r^2+a^2)}$

$\vec{E}_- = \frac{q}{4\pi\varepsilon_0}\frac{1}{(r^2+a^2)}$

The total electric field

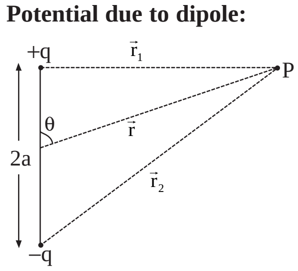
$E = (E_+ + E_-)\cos\theta$

$E = \frac{2qa}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$

$\vec{E} = -\frac{\vec{p}}{4\pi\varepsilon_0(r^2+a^2)^{3/2}}$

"–ve" sign indicates, the direction of electric field on equatorial plane is opposite to the direction of dipole moment vector $\vec{P}$.

- **Potential due to dipole:**



Potential at point P is V.

$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r_1} - \frac{q}{r_2}\right)$

$r_1^2 = r^2 + a^2 - 2ar\cos\theta$
$r_2^2 = r^2 + a^2 + 2ar\cos\theta$

Using Binomial approximation:

$\frac{1}{r_1} = \frac{1}{r}\left[1 + \frac{a}{r}\cos\theta\right]$

$\frac{1}{r_2} = \frac{1}{r}\left[1 - \frac{a}{r}\cos\theta\right]$

$V = \frac{q}{4\pi\varepsilon_0}\frac{2a\cos\theta}{r^2}$

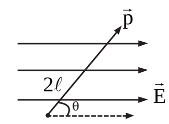
$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$ (r >> a)

Torque on a Dipole in an Electric field:

Dipole experiences zero force when it is placed in uniform electric field, since each charge experience a force equation in opposite directions.

The dipole experiences torque since line of action of two forces is not same.

Torque = $2q\ell\sin\theta\times E$



In vector form: Torque $= \vec{p}\times\vec{E}$

Potential Energy of a Dipole in an Electric Field:

$U = -\vec{p}\cdot\vec{E}$

Force on a dipole in non-uniform electric field $\vec{E}$:

$\vec{F} = p\frac{\partial\vec{E}}{\partial r}$, where $\frac{\partial\vec{E}}{\partial r}$ is the derivative of electric field with respective to distance along the direction of the dipole.

Important IPE Questions

Very Short Answer Questions

1. What is meant by the statement 'charge is quantized'?
2. How many electrons constitute 1 C of charge?
3. What happens to the force between two charges if the distance between them is (a) halved, (b) doubled?

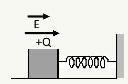
Short Answer Questions

1. State and explain Coulomb's inverse square law in electricity.
2. Derive the equation for the couple acting on a electric dipole in a uniform electric field.
3. Derive an expression for potential at a distance r from the centre of a dipole.

JEE Model Questions

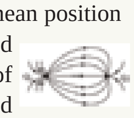
1. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side 'a'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where 'k' is a constant. Then N is..

2. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge +Q on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be:



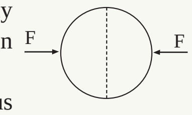
a) of the same frequency and with shifted mean position
b) of the same frequency and with the same mean position
c) of changed frequency and with shifted mean position
d) of changed frequency and with the same mean position

3. A few electric field lines for a system of two charges Q_1 and Q_2 fixed at two different points on the x-axis are shown in the figure. These lines suggest that:



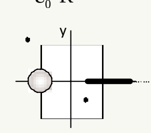
a) $|Q_1| > |Q_2|$ b) $|Q_1| < |Q_2|$
c) at a finite distance to the left of Q_1 the electric field is zero
d) at a finite distance to the right of Q_2 the electric field is zero

4. A uniformly charged thin spherical shell of radius R carries uniform surface charge density of σ per unit area. It is made of two hemispherical shells, held together by pressing them with force F (see figure). F is proportional to:



a) $\frac{1}{\varepsilon_0}\sigma^2R^2$ b) $\frac{1}{\varepsilon_0}\sigma^2R$
c) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R}$ d) $\frac{1}{\varepsilon_0}\frac{\sigma^2}{R^2}$

5. A disk of radius a/4 having a uniformly distributed charge 6C is placed in the x-y plane with its centre at (–a/2, 0, 0). A rod of length a carrying a uniformly distributed charge 8C is placed on the x-axis from x = a/4 to x = 5a/4. Two point charges –7C and 3C are placed at (a/4, –a/4, 0) and (–3a/4, 3a/4, 0), respectively. Consider a cubical surface formed by six surfaces $x = + a/2$, $y = + a/2$, $z = + a/2$. The electric flux through this cubical surface is:



a) $(-2C)/\varepsilon_0$ b) $2C/\varepsilon_0$
c) $10C/\varepsilon_0$ d) $12C/\varepsilon_0$

Answers

1) 3 2) a 3) a, d 4) a 5) a